Multi-objective hybrid harmony search-simulated annealing for location-inventory-routing problem in supply chain network design of reverse logistics with CO_2 emission

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Abstract. The advancement of supply chain network design in reverse logistics is gaining interest from the industries. In recent years, the multi-objective framework of the problem has been widely studied by researchers. This paper integrates three different levels of decision planning in supply chain network design: location-allocation problem for strategic planning, inventory planning management for tactical planning, and vehicle routing for operational planning. A location-inventory-routing problem based on the economic production quantity model with environmental concerns is considered. This study aims to minimise the total cost of operating facilities, inventory and distance travelled by the vehicles as the first objective while minimising the CO_2 emission cost as the second objective. Due to the complexity of the problem, a non-dominated sorting and ranking procedure is applied into a Multi-Objective Hybrid Harmony Search-Simulated Annealing (MOHS-SA) algorithm to find the trade-off between these two objectives. Computational experiments on the benchmark instances indicate that the proposed MOHS-SA algorithm can produce well-distributed Pareto-optimal solutions for multi-objective problems.

1. Introduction

Traditionally, supply chain network design focuses more on the economic value rather than the other factor such as environmental issue. Recently, the eco-friendly supply chain that considered CO_2 emission in reverse logistics has been studied widely. Many developed countries are very concerned about reducing their carbon emission. Hence, both environmental effect and economic strategy need to be integrated effectively and efficiently in the reverse logistics of the supply chain to improve the environmental conditions as well as the structure of an organisation's network design. Transportation activities are one of the significant sources of air pollution and greenhouse emission [1]. Long travel distances lead to the increasing of vehicle emission on the transportation routes [2]. Thus, the reduction in the travelled distance could play a significant role in reducing the carbon footprint. The gas emission would be minimised according to the distance travelled by the vehicle and also depending on the weight load of the vehicle at each route. Besides that, network design is the main focus when concerning the green supply chain. The main critical issues in the supply chain network are dealing with the strategic location and allocation of the customers, the inventory planning and the routing of vehicle delivery. These problems have to be solved separately due to their complexity. However, these problems are connected and will affect the optimal cost in the whole system of the supply chain. Therefore, we are motivated to study the green supply chain especially in the reverse logistics of the location-inventory-routing problem (LIRP) where the environmental protection needs to be determined along with the process of network design.

Several studies are found to be related to the multi-objective of LIRP. The reliable model for LIRP which consider the disruption risks is studied by [3]. They minimised the total cost as well as the total failure costs related to the disruption centre and solved using Archived Multi-objective Simulated Annealing (AMOSA). In their inventory system, partial backorder is used when dealing with the occurrence of stock out and the algorithm parameters are tuned by using the Taguchi method. The second-generation genetic algorithm is used to solve the multi-objective LIRP that minimise the total cost and transit time [4]. The results showed that the lead time give more impact on warehouse planning as compared to the minimum order quantity constraint. By using the Taguchi method for a set of parameter tuning in [3], [5] compared the Multi-objective Particle Swarm Optimisation (MOPSO) and NSGA-II to obtain the trade-off between cost and impact pollution via CO_2 emission. The study showed that the NSGA-II is more powerful in solving the design of algae biofuel production and distribution network. A LIRP with distribution outsourcing (LIRPDO) and time windows constraint is solved by [6] and [7], respectively. [6] used General Variable Neighbourhood Search (GVNS) based on a meta-heuristic algorithm since the exact solution are limited to the small problems only. Meanwhile, [7] proposed an Improvised Multi-objective Ant Colony Optimisation (IMACO) and compared it with MACO to optimise the location cost, inventory cost, transportation cost, and penalty cost simultaneously. They improvised the method by adding a crossover operator for global search ability and ant state transition rules to avoid the local optimum trap. It is proven that IMACO outperformed the solutions in MACO.

Due to the time complexity of the multi-objective problem, an exact method is not an appropriate approach to be used. Thus, most of the problems have been solved by metaheuristic approaches. Over the years, some literature on multi-objective harmony search (MOHS) algorithms are found. [8] proposed multi-objective for optimal power point flow (OPP) problem with fuel cost and real power loss minimisation. They used fast elitist non-dominated sorting and crowding distance to find Pareto optimal in the MOHS algorithm. Followed by [9], they minimised the fuel cost and emission for environmental/economic dispatch (EDD) problem. The dynamic crowding distance is applied in the MOHS algorithm. Both [8] and [9] used the standard HS into their multi-objective problem. An improved HS that incorporate with niche technique is introduced by [10]. The Pareto set filter and an elite individual preserving are added to the algorithm to solve the optimal design of the groundwater remediation system. They minimised the remediation cost and the mass remaining in aquifers as well. The minimisation of total waiting time and total flow time of flow line and manufacturing cell scheduling problem (FMCSP) has been studied by [11]. They used a one-point crossover and applied it for diversification of HS. [12] proposed an adaptive formula of PAR, bandwidth and HMCR to update the adjusting parameter of HS. They obtained minimum total network cost and total network emission.

In addition, [13] solved the scheduling project with time and cost minimisation using MOHS. They used a novel stochastic derivative in HS and compared it with a GA and ant colony. It is shown that the solution in HS obtained a good Pareto solution. Binary-coded multi-objective optimisation problem has been introduced by [14] and [15]. In [14], the pitch adjustment operator is modified. The proposed multi-objective binary harmony search (MBHS) outperformed the convergence metric and diversity metric in NSGA-II. [15] used binary coded in MOHS to represent the urban road's directions. They used non-dominated sorting and crowding distance in their binary HS method. Multi-objective for facility location problem and location-allocation problem have been solved by [16] and [17], respectively. [16] combined MOHS with the grouping encoding procedure and non-dominated sorting procedure while [17] used the Taguchi method to calibrate the parameters for all algorithm to be compared. [18] proposed new memory considering rule and dynamic values of HMCR and PAR in niching multi objective harmony search (NMOHS) to solve multimodal multi objective optimization problem. Their proposed method perform much better than the other existing multimodal multi objective algorithm. In [19], HS was modified to minimize two objectives simultaneously, used tool life and operation time for manufacturing lean. The numerical analysis shows that the proposed method is more accurate and effective when comparing with standard HS and GA. Integrated approach of location decisions, routing and allocation to the distribution network problem are solved by [20]. Multi objective harmony search with parameter adjusting by Taguchi method was proposed and the performance are comparable with the NSGA-II and NRGA.

From the stated literature above, none of them solved the multi-objective location-inventory-routing problem (LIRP) that considered the Economic Production Quantity (EPQ) model with CO_2 emission. In this study, we proposed a Multi-Objective Harmony Search-Simulated Annealing (MOHS-SA) algorithm that hybrid the idea of annealing in SA into the HS to reserve the solution in the harmony memory (HM) during the improvisation process. This technique is proven to reduce the possibility of the solution getting easily trapped into local optima. Fast non-dominated sorting and crowding distance are implemented for obtaining the Pareto front among the solutions in HM. The fuzzy membership approach is calculated to obtain the best compromise solution in the first front to determine the best trade-off between cost minimisation and CO_2 emission. The remainders of the paper are organised as follows: the details of the LIRP with the mathematical model are explained in Section 2. The proposed MOHS-SA for LIRP is presented in Section 3. The algorithm is tested on several benchmark instances and the results are analysed in Section 4. The last section gives the conclusions of the study.

2. Location-inventory-routing problem

LIRP is a problem that integrates the facility location problem with the inventory planning and routing of vehicles. The main decisions in this problem are to determine the number of depots and their respective locations to be opened, the optimal production quantity to meet all the customer demands without shortage and the shortest distance travelled by the vehicles without violating the vehicle capacity limit.

In this study, two objectives are solved. In the first objective, the total fixed operating cost, inventory cost, as well as distance cost, travelled by vehicles are to be minimised. In the second objective, the environmental effect of the CO_2 emission is to be minimised. The assumptions that we made into the model are, the demand and returns items are known and constant and the weight of each vehicle are the same since the vehicles are homogeneous. The total payload is depending on the carrying demands and returns by the vehicle on that route. Because of the complexity and also the difficulties in controlling the behaviour of the driver and other factors such as the situation of the traffic and the road conditions, the value of emission factor is also set to be a constant value (see, Section 4).

2.1. Mathematical modeling
The mathematical formulation of multi-objective LIRP is described as follows:

Set:
I = sets of all depots $(i = 1, 2, \dots, I)$
J = sets of all customers $(j = 1, 2, \dots, J)$
K = sets of all vehicles $(k = 1, 2, \dots, K)$

Input parameter:
D_j = demand of customer j
R_j = non-defect items return by customer j
S_j = defect items return by customer j
d_{ij} = distance from i to j
$Vehc_k$ = capacity of vehicle k
N = number of customer
F_i = fixed operating cost of depot i
DC = distance cost per miles
$Vehd_i$ = maximum throughput at depot i
KC = cost of setup production
h = holding cost per unit inventory
P = production rate per batch
TL_{ijk} =total load of vehicle k at depot i through customer j
W = weight of vehicle in tons
EF = emission factor

Decision variables:
U_{lk} = auxiliary variable for sub-tour elimination constraints in vehicle k of customer l.
$$z_i = \begin{cases} 1, & \text{if depot } i \text{ is open} \\ 0, & \text{otherwise.} \end{cases}$$
$$y_{ij} = \begin{cases} 1, & \text{if depot } i \text{ assign to customer } j \\ 0, & \text{otherwise.} \end{cases}$$
$$x_{ijk} = \begin{cases} 1, & \text{if arc } (i,j) \text{ is traveled by vehicle } k \\ 0, & \text{otherwise.} \end{cases}$$
Q_i = optimal number of production quantity for each depot i

$$\min \quad f_1 = \sum_{i\in I} F_i z_i + \sum_{i\in I\cup J}\sum_{j\in I\cup J}\sum_{k\in K} DC \times d_{ij}x_{ijk} + \frac{1}{2}h\sum_{i\in I} Q_i\left(1 - \frac{\sum_{i\in I}\sum_{j\in J}(D_j + R_j + S_j)y_{ij}}{P}\right) + \frac{\sum_{i\in I}\sum_{j\in J} Kc(D_j - R_j + S_j)y_{ij}}{\sum_{i\in I} Q_i} \tag{1}$$

$$f_2 = \sum_{i\in I\cup J}\sum_{j\in I\cup J}\sum_{k\in K} W \times EF \times D_j d_{ij} x_{ijk} \tag{2}$$

subject to:
$$\sum_{k\in K}\sum_{i\in I\cup J} x_{ijk} = 1, \forall j \in J \tag{3}$$
$$\sum_{i\in I}\sum_{j\in J} x_{ijk} \leq 1, \forall k \in K \tag{4}$$
$$\sum_{i\in I\cup J}\sum_{j\in J} D_j x_{ijk} \leq Vehc_k, \forall k \in K \tag{5}$$
$$\sum_{i\in I\cup J}\sum_{j\in J} (R_j + S_j)x_{ijk} \leq Vehc_k, \forall k \in K \tag{6}$$
$$TL_{ijk} - (D_j - (Rj + S_j))x_{ijk} \leq Vehc_k, \forall i \in I, \forall j \in J, \forall k \in K \tag{7}$$

$$\sum_{i \in I \cup J} x_{ijk} - \sum_{j \in I \cup J} x_{jik} = 0, \forall i \in I, \forall k \in K \tag{8}$$

$$\sum_{j \in J} D_j y_{ij} \leq Vehd_i z_i, \forall i \in I \tag{9}$$

$$U_{lk} - U_{jk} + N x_{ljk} \leq N - 1, \forall l, j \in J, \forall k \in K \tag{10}$$

$$\sum_{u \in I \cup J} x_{iuk} + x_{ujk} - y_{ij} \leq 1, \forall i \in I, \forall j \in J, \forall k \in K \tag{11}$$

$$y_{ij} \in \{0,1\}, \forall i \in I, \forall j \in J \tag{12}$$

$$x_{ijk} \in \{0,1\}, \forall i \in I \cup J, \forall j \in I \cup J, \forall k \in K \tag{13}$$

$$z_i \in \{0,1\}, \forall i \in I \tag{14}$$

$$U_{lk} \geq 0, \forall l \in J, \forall k \in K \tag{15}$$

$$Q_i \geq 0, \forall i \in I \tag{16}$$

The objective of LIRP in equation (1) is to minimise the total fixed operating cost of depots, cost of inventory and the total distance cost travelled by the vehicles. The second objective in equation (2) represents the CO_2 emission minimisation. Constraints in equation (3) and equation (4) indicate that each of the customers has to be assigned in a single route and it can be served by only one vehicle. The constraint of total demand at each route that cannot exceed the vehicle capacity limit is shown in equation (5). The capacity of vehicle limit that consider the return items and the total payload for each route through the respective customers are also needed to be fulfilled. These constraints are shown in equation (6) and equation (7) respectively. To make sure the vehicle must start and end at the same depot, constraint in equation (8) is added. Besides the vehicle capacity limit, the capacity constraint for the depot is given in equation (9). Equation (10) represents the new sub tour elimination constraint and equation (11) specified that the customer will be assigned to the depot if there is a route from that depot. The binary values on the decision variables and the positive values for the auxiliary variable and optimal production quantity are defined in equation (12)–(16), respectively.

3. Multi-objective harmony search - simulated annealing algorithm

Harmony Search (HS) is a popular-based metaheuristic algorithm that imitates the music improvisation of a group orchestra [21]. There are three basic techniques of harmony improvisation have been practiced by the musician namely: (1) select any pitch that has been played from the previous harmony memory, (2) fix the pitch that sound similar to any previous pitch in the memory, or (3) compose a new music harmonisation. The procedure of searching a perfect state of harmony is similar to the process of obtaining the optimal solution in the optimization problem. Over the years, there are many modifications to the HS by the researchers, especially when dealing with the LIRP problems. In this paper, the modifications that have been implemented in the HS algorithm are detailed below.

Step 1: Initialise HM and Setting Parameter

Harmony memory initialisation is the first stage to build a set of solution vectors in a population. A set of solution vector can be generated either randomly or by a heuristic. In the proposed MOHS-SA of LIRP, the solution vector in HM represent the sequence of customers at each depot and routes and the fitness values for each objective. The number of solutions in the HM is the harmony memory size (HMS). The size of HM should be defined adequately so as the choices of solution vector in HM is good enough for the next selection of the improvisation process. HM can be described in the following matrix:

$$HM = \left[\begin{array}{cccc|ccc} x_1^1 & x_2^1 & \cdots & x_n^1 & f_1(x^1) & \cdots & f_n(x^1) \\ x_1^2 & x_2^2 & \cdots & x_n^2 & f_1(x^2) & \cdots & f_n(x^2) \\ \vdots & \vdots & \ddots & \vdots & \vdots & \cdots & \vdots \\ x_1^{HMS} & x_2^{HMS} & \cdots & x_n^{HMS} & f_1(x^{HMS}) & \cdots & f_n(x^{HMS}) \end{array} \right], \tag{17}$$

where,
n = number of objective function,
x_i^j = decision variable for $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots,$HMS,
$f_n(x^j) = n^{th}$ fitness function of x^j for $j = 1, 2, \ldots,$HMS.

The parameters setting used in the proposed HS algorithm are HMCR, PAR, HMS, and the stopping criterion. An appropriate value of HMCR will lead to the choices of good solutions as an element of new solutions. Too high of HMCR will limit the exploration space of the solution, but if too low, the selected solution for the next iteration is based on a few good solution only. The same goes for PAR, a higher value of PAR is not recommended as the solution easily getting stuck at the local optima with a higher value of PAR. Meanwhile, the speed of convergence will decreased if the value of PAR is too small. Therefore, to use the memory effectively, the value of HMCR should be between 0.7 and 0.95 and the value of PAR is supposed to be between 0.1 and 0.5 [22]. However, this range may be suitable and valid for some problems only.

Step 2 : Improvisation process
The improvisation process consists of two phases namely the construction phase and the adjustment phase. A standard HS utilise a single search solution to evolve. This makes the algorithm converged slowly to the optimal solution. To increase the speed of convergence, several new solutions are generated at each iteration and called HM$_{new}$. Each solution in the HM has the same probability to be chosen as a new solution [23].

According to a standard HS, a new set of solution vector will be created outside of HM if the randomly generated number is greater than the value of HMCR. Otherwise, one of the solution vector set will be selected within the HM and will be improvised using one of the local search techniques if the random number is less than PAR. Here, the values of HMCR and PAR are very important to control the diversification and intensification of the algorithm. To be more effective, we proposed the dynamic HMCR and PAR as has been introduced by [24]. They introduced the formulation of dynamic HMCR and PAR and the changes will be done at each iteration without condition. But in this study, the formulation will be reduced gradually if there is no improvement found in the solution. The following formulation of the proposed dynamic HMCR and PAR are shown below:

$$HMCR_{it} = HMCR_{max} - (HMCR_{max} - HMCR_{min})\frac{it}{MaxIt}, \tag{18}$$

$$PAR_{it} = PAR_{max} - (PAR_{max} - PAR_{min})\frac{it}{MaxIt}, \tag{19}$$

where,
it = the current iteration,
$MaxIt$= the maximum iterations,
$HMCR_{max}$ = the maximum value of the HMCR,
$HMCR_{min}$ = the minimum value of the HMCR,
PAR_{max} = the maximum value of the PAR,
PAR_{min} = the minimum value of the PAR.

In this problem, the range values of HMCR and PAR are set to be [0.7,0.99] and [0.1,0.6] respectively. The reason for reducing the HMCR slowly is to increase the probability of exploring more solution space, not in the HM. Hence, the global optimal can be attained [25]. The dynamic value of HMCR and PAR can avoid the solutions getting trapped in the local optimum quickly.

Local Neighbourhood Search
To increase the intensification of the method, the proposed MOHS-SA implements the multi-local neighbourhood search as proposed in [26]. Several techniques of local search that consider the replacement of customer's sequence within the same depot, same vehicle route and also between any two depots have been implemented into the proposed MOHS-SA. At each generated new solution in HM_{new} for each iteration, once the condition of adjustment phase is met, one of the local search techniques listed below is applied:

Swap: exchange the position of two random customers within the same route or between different routes. Swapping can be done within the same depot or between the depots.
Insertion: insert a customer in between two other random customers within the same depot.
Relocation: relocate the customers from the current depot to a new depot.
2-opt: swapping two customers in the same vehicle route and reverse the substring between the swapped customers.
3-opt: deleting three edges of the customers in the same vehicle route and create another three sub tours in three possible ways: non-reversing substring, with one reversing substring or with two reversing substring.

Swapping within a depot and insertion are two techniques that involve the searching of the neighbourhood within the same depot while swapping between two depots and relocation are techniques that require two different depots. When the neighbourhood is searched inclusive among customers in the same vehicle route, the 2-opt and 3-opt are considered.

Simulated Annealing
As mentioned before, simulated annealing which has been introduced earlier by [27] was hybridised into our proposed method. The framework of SA that consider the worst solution to be kept in the HM for the next selection is implemented along the process of improvisation with a certain probability. By preserving the worst solution in a population, it provides practical randomness into the search to avoid the local extreme points. The probability of acceptance in SA is depending on the value of cooling rate, annealing temperature and the difference in the value of an objective function. The annealing temperature will be reduced once the worst of new solution is detected. In our problem, we set the initial temperature and cooling rate as 30^0C and 0.98 respectively. We found that, the higher value in the cooling rate, the slower the temperature reduction.

Step 3: Update HM
At each iteration, the solution vector in the HM will be updated. At first, the solutions in the current HM will be combined with the solutions in the HM_{new}. Then, the set of solutions to be kept for the next improvisation process with the size of HM will be selected and ranked according to the best solutions provided by the non-dominated sorting procedure and crowding distance calculation.

Fast non-dominated sorting and ranking procedure
For a single objective, the solution vectors in the HM are ranked according to the fitness value.

In the minimisation problem, the lowest fitness values will be ranked first. However, since this paper is dealing with the multi-objective, the mentioned approach is not appropriate. Therefore, a fast non-dominated sorting and ranking procedure introduced by [28] is used to rank the Pareto front in the multi-objective solution. The definition of non-dominated ranking is explained as follows:

For minimisation problem, x_2 is said to be dominated by x_1 if the following conditions are satisfied:
(i) $f_i(x_1) \leq f_i(x_2) \forall i \in \{1, 2, ..N\}$,
(ii) $f_j(x_1) < f_j(x_2) \forall j \in \{1, 2, ..N\}$.

Since x_1 dominates x_2, x_1 is called as the non-dominated solution. The non-dominated solution of the entire space is called a Pareto optimal solution. Generally, if x_1 is a non-dominated solution, at least one of the objective of solution x_1 must be better than x_2 and cannot be worse than the solution in x_2.

In the non-dominated sorting approach, each solution is compared with all other solutions in the population and being assigned to the first front if that solution is not dominated by others. This approach contains many redundant comparisons, hence the fast non-dominated solution approach is proposed. In this approach, the comparison between any two solutions is performed only once. This will greatly reduce the time complexity.

Crowding Distance
Identifying the worst solution in HM for multi-objective is not as simple as in a single objective. The worst solution in the last front is replaced with a better solution according to their crowding distance value which is also introduced by [28]. The solution that obtained the least crowding distance compared to others defined as a better solution. The formulation of crowding distance is shown in equation (20) :

$$CD_i = CD_i + \frac{f^r_{i+1} - f^r_{i-1}}{f^r_{max} - f^r_{min}}, \tag{20}$$

where,
r= number of objective functions,
f^r_{i+1}= rth objective of $i+1$ individuals,
f^r_{i-1}= rth objective of $i-1$ individuals,
f^r_{max}= maximum value of rth objective,
f^r_{min}= minimum value of rth objective.

Fuzzy Membership Approach
In a multi-objective optimisation problem, the first front is represented as a set of best solution vector in HM. This is called a set of Pareto optimal. Among all solutions in Pareto optimal, the best-compromised solution using the fuzzy membership approach is chosen as proposed by [8]. The value of fuzzy membership for each function μ_i^k is defined as:

$$\mu_i^k = \begin{cases} 1, & f_i \leq f_i^{min} \\ \frac{f_i^{max} - f_i}{f_i^{max} - f_i^{min}}, & f_i^{min} < f_i < f_i^{max}, \\ 0, & f_i \geq f_i^{max} \end{cases} \tag{21}$$

where,
f_i^{min}= minimum value of ith objective function among all non-dominated solutions,
f_i^{max}= maximum value of ith objective function among all non-dominated solutions.

Hence, the normalised value of fuzzy membership μ^k is calculated as:

$$\mu^k = \frac{\sum_{i=1}^{N} \mu_i^k}{\sum_{k=1}^{P} \sum_{i=1}^{N} \mu_i^k}, \tag{22}$$

where,
$P=$ total number of non-dominated solution at first front,
$N=$ number of objective function.
The solution that has the highest value of μ^k will be selected as the best compromised solution of that iteration.

Step 4: Stopping criteria
The trade-off solution between these objective functions will be obtained once the iteration at the first objective reaches the 100 consecutive non-improving solutions. The maximum number of iterations is also been considered as a stopping criterion.

The proposed MOHS-SA is the extension of HS-SA in [29] with the inclusion of a multi-objective framework procedure. We consider both economic impact and environmental effect in the objective functions. The algorithm of the proposed MOHS-SA is given in **Algorithm 1**.

Algorithm 1: Proposed MOHS-SA

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begin
Initialisation phase:
Define harmony vectors and fitness function for each objective
Define parameter setting: HMCR$_{max}$, HMCR$_{min}$,PAR$_{max}$,PAR$_{min}$, HMS, MaxIt, T_{it},α
Generate initial population of HM
Improvisation phase:
Do the route-allocation and inventory planning
while (stopping criteria is not met) **do**
  while (no. of harmony vectors < size of HM$_{new}$) **do**
    Calculate HMCR$_{it}$ and PAR$_{it}$
    if (rand $\leq$ HMCR$_{it}$) then
      choose solution from the HM randomly
      if (rand $\leq$ PAR$_{it}$) then
        perform one of the local neighborhood search for location-inventory-routing
        $\Delta 1 = f_1(x') - f_1(x)$
        $\Delta 2 = f_2(x') - f_2(x)$
        if ($\Delta 1 < 0$ and $\Delta 2 < 0 \,\|\, \Delta 1 < 0$ and $\Delta 2 = 0 \,\|\, \Delta 1 = 0$ and $\Delta 2 < 0$)
          $x \longleftarrow x'$
        else
          if (rand $< exp(\frac{-\Delta 1}{T_{it}})$)
            $x \longleftarrow x'$
            $T_{it+1} = \alpha \times T_{it}$
          end if
        end if
      else
        keep the harmony vectors
      end if
    else
      explore the other harmony vectors space
    end if
    Calculate the new fitness function of each harmony vector for both objectives
  end while
  Combine the HM with HM$_{new}$. Sort the fitness function using fast nondominated sorting
  Perform crowding distance to update the best harmony vector with size of HMS
  $it = it + 1$
end while
Return the best solution vectors of harmony = best compromise solution in first front using fuzzy membrship function
end

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4. Results and discussion

The MOHS-SA is implemented in MATLAB software R2017b on a laptop computer with 1.60GHz Intel Core i5-4200U CPU with 8GB of RAM and tested with benchmark instances of Perl, Gaskell and Christofides. Since these benchmark data are specifically for the location-routing problem (LRP), the additional data for return rate and production rate were generated randomly by using a uniform distribution. The types of return items can be either non-defect or defect, which is denoted by R_j and S_j respectively. We set the number of returns should be always less than the demand at each customer where $Tr_j = R_j + S_j$ and $Tr_j \sim U(0, D_j)$. 70% of the returns are non-defect and the remaining are defect items. The production rate is set to be greater than the total demand and returns including both defect and non-defect, where $P > \sum_{j\in J}(D_j + R_j + S_j)$ to avoid the shortage in the inventory. The characteristics of the instances are given in table 1.

Table 1. Perl, Gaskell and Christofides instances.

Instances	Customer	Depot	Depot Capacity	Vehicle Capacity
Perl 1	12	2	280	140
Perl 2	55	15	550	120
Perl 3	85	7	850	160
Gaskell 1	21	5	15000	6000
Gaskell 2	22	5	15000	4500
Gaskell 3	29	5	15000	4500
Gaskell 4	32	5	35000	8000
Gaskell 5	36	5	15000	250
Christofides 1	50	5	10000	160
Christofides 2	75	5	10000	140
Christofides 3	100	5	10000	200

To assess the performance of the MOHS-SA algorithm, it has been compared with the standard MOHS. The weakness of HS which is easily getting trapped in local optima is improved by implementing the dynamic parameters of HMCR and PAR, together with multi-local search neighbourhood and the idea of SA. In SA, there is a chance of the worst solution to be accepted for a certain probability. Besides, this problem is not just to optimise the cost but also the emission from CO_2. As mentioned earlier, the gas emission is depending on the distance travelled, the weight, and the emission factor of the vehicle. The formulation of CO_2 emission on the second objective function is given as follows:

$$\text{CO2 emission} = \text{distance (in km)} \times \text{weight (in tons)} \times \text{emission factor (in kg/tons-km)}.$$

In this study, Volvo engines have been chosen since it complies with the legal requirements and many engines have even been introduced a couple of years before the legal requirements have come into force. Volvo has converted the certification values into emission per litre of fuel. The information of all related data can be found via the link $https://www.volvotrucks.com/content/dam/volvo/volvo-trucks/markets/global/pdf/our-trucks/Emis_eng_10110_14001.pdf$. In our problem, truck for distribution traffic is selected. Based on the data given, the average fuel consumption for this particular type of vehicle is about 0.275 litre for each km and the full payload of the truck is equal to 8.5 tons. Due to the complexity of considering the changes in the load each time of the delivery of the products, it is assumed that the vehicle has the same emission factor for each route delivery and it is constant. The calculation of the emission factor for the distribution truck is obtained as follows:

$$\text{emission factor} = \text{fuel consumption} \times \text{combustion of carbon dioxide.}$$
$$=0.275l/km \times \frac{2.6kg/l}{8.5tons}$$
$$=0.0841kg/tons - km.$$

Table 2. Best out of five compromise solutions of Perl, Gaskell and Christofides dataset for MOHS and MOHS-SA.

Problem	MOHS			MOHS-SA		
Instance	cost	CO_2	μ^k	cost	CO_2	μ^k
Perl 1	**252.5160**	**62.1482**	0.2533	**252.5160**	**62.1482**	**0.3333**
Perl 2	1530.0153	372.3902	0.4822	**1392.2623**	**292.6311**	**0.5178**
Perl 3	2289.6487	581.9989	0.1226	**1996.9371**	**399.1504**	**0.1755**
Gaskell 1	1570.0904	208.2502	0.0041	**1556.0930**	**208.0378**	**0.0065**
Gaskell 2	1001.5455	**251.4397**	0.0061	**999.3237**	441.4444	**0.0065**
Gaskell 3	957.9846	419.9171	0.4894	**782.8403**	**331.5715**	**0.5106**
Gaskell 4	1133.0476	406.8969	0.4975	**1036.4327**	**358.2591**	**0.5025**
Gaskell 5	780.2885	470.7335	0.1451	**547.5508**	**281.2442**	**0.1686**
Christofides 1	851.2401	**350.2869**	0.0025	**825.2162**	374.7012	**0.0047**
Christofides 2	1315.0926	735.4131	0.4928	**1159.8065**	**637.1060**	**0.5072**
Christofides 3	**1289.0992**	543.2531	0.4950	1302.9838	**531.7702**	**0.5060**

Since this is a multi-objective problem, a trade-off between the cost and the CO_2 emission needs to be identified. For each of the problem instances, five runs of experiments are performed in MATLAB for each algorithm. The values of the first and second objectives for each run are recorded together with their values of fuzzy membership function μ^k. The highest value of μ^k is indicating the best combination of cost function and CO_2 function. Among the five runs, the highest μ^k is selected as the best trade-off solution. Table 2 represents the best compromise solution among the five runs at each problem instances for MOHS and MOHS-SA with their respectively fuzzy membership function values. It is worth noting that the number of customers and depots are small in Perl 1, hence both algorithms managed to find the same best compromise solution. As compared to the MOHS, the MOHS-SA outperformed the solutions for most instances in Perl, Gaskell and Christofides for both objectives. However, some results show the MOHS can compete with the MOHS-SA. As can be seen in table 2, MOHS obtained the minimum value in the first objective for Christofides 3, and in the second objective for Gaskell 2 and Christofides 1. This means that the MOHS-SA strictly dominates the solutions in MOHS for all problem instances except Gaskell 2, Christofides 1 and 3. This is the reason why the values of μ^k are very important to be determined. It is purposely for obtaining a better trade-off solution. Based on the values of μ^k, the MOHS-SA shown the best method to be applied since all of the fuzzy membership function values obtained in table 2 are greater than MOHS. To illustrate the results of MOHS-SA further, figure 1–10 shows the Pareto front found in five runs analysis of each problem instances for Perl, Gaskell and Christofides, except Perl 1 where each run of the algorithm produced the identical solution (which eventually converged to only one point in the Pareto front).

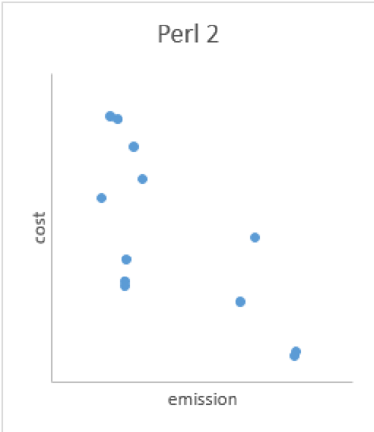

Figure 1. Pareto front in Perl 2 instance.

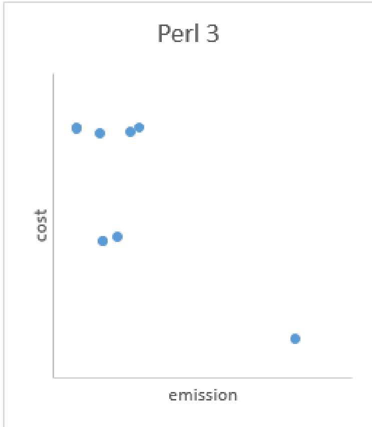

Figure 2. Pareto front in Perl 3 instance.

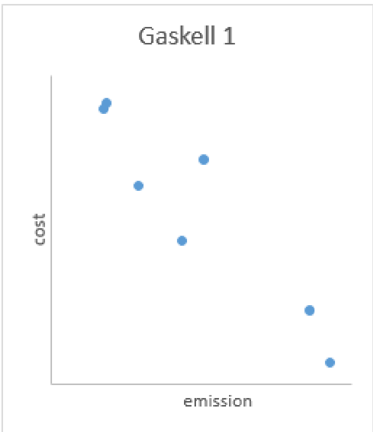

Figure 3. Pareto front in Gaskell 1 instance.

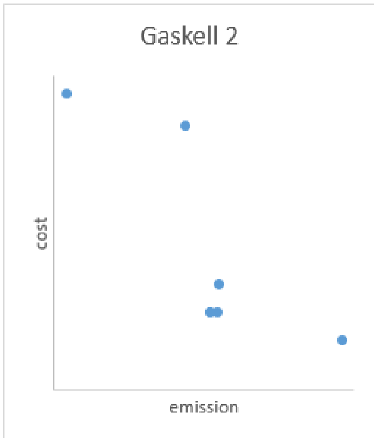

Figure 4. Pareto front in Gaskell 2 instance.

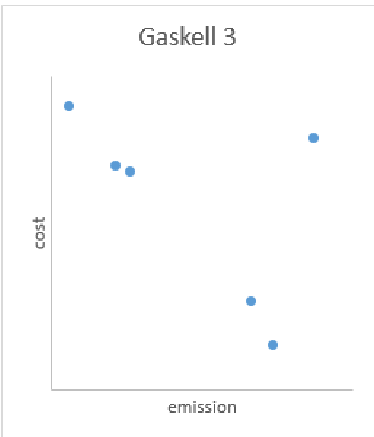

Figure 5. Pareto front in Gaskell 3 instance.

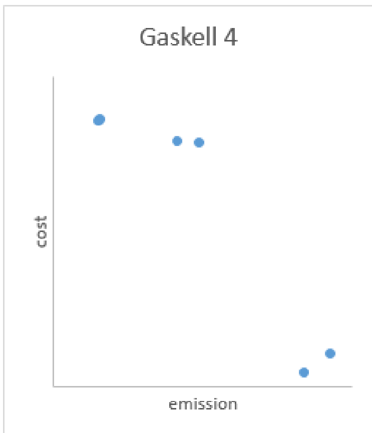

Figure 6. Pareto front in Gaskell 4 instance.

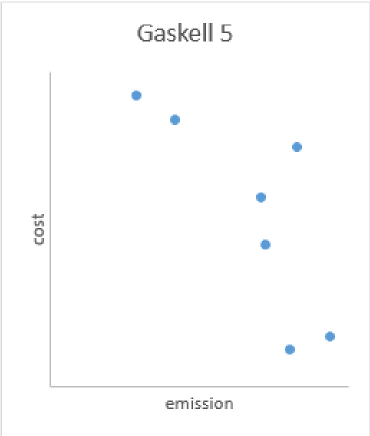

Figure 7. Pareto front in Gaskell 5 instance.

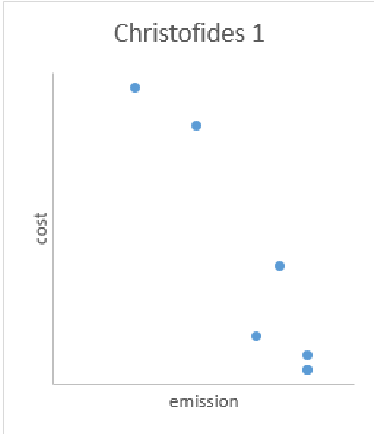

Figure 8. Pareto front in Christofides 1 instance.

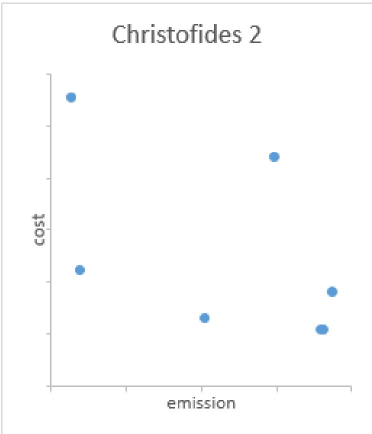

Figure 9. Pareto front in Christofides 2 instance.

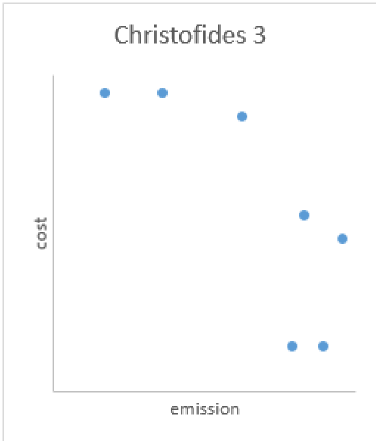

Figure 10. Pareto front in Christofides 3 instance.

5. Conclusions

In this paper, a MOHS-SA is proposed for solving the LIRP which considers the EPQ model and CO_2 emission. In the proposed algorithm, the multi solutions are generated and multi neighbourhood search techniques are implemented during the process of improvisation. Both HMCR and PAR are changed dynamically when there is no improvement found in the solution. To increase the intensification capability, the hybridisation of HS and SA is performed. The worst solution that is possible to be accepted for the next selection of improvisation, is kept in the HM to avoid the premature solution. Both defect and non-defect items are considered in the model formulation of inventory. A fast non-dominated sorting and crowding distance procedure is implemented to solve the multi-objective problem. Best compromise solutions among the methods have been identified using the fuzzy membership approach. Three well-known benchmark dataset of Perl, Gaskell and Christofides are used to evaluate the proposed algorithm. The results obtained show that the proposed MOHS-SA is efficient to be implemented in LIRP and successful in obtaining a better Pareto front for multi-objective problem compared to the MOHS. In future research, the social impact could be considered in the multi-objective problem.

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References

[1] Wang F, Lai X and Shi N 2011 A multi-objective optimization for green supply chain network design *Decision Support Systems* **51(2)** 262–269

[2] Elhedhli S and Merrick R 2012 Green supply chain network design to reduce carbon emissions *Transportation Research Part D: Transport and Environment* **17(5)** 370–379

[3] Rayat F, Musavi M and Bozorgi-Amiri A 2017 Bi-objective reliable location-inventory-routing problem with partial backordering under disruption risks: a modified AMOSA approach *Applied Soft Computing* **59** 622–643

[4] Di Y L, Zhou Y J and Liang X Y 2018 Multi-objective and multi-product inventory-location model under stochastic demand *DEStech Transactions on Social Science, Education and Human Science, (icssd)*

[5] Asadi E, Habibi F, Nickel S and Sahebi H 2018 A bi-objective stochastic location-inventory-routing model for microalgae-based biofuel supply chain *Applied Energy* **228** 2235–2261

[6] Karakostas P, Sifaleras A and Georgiadis M C 2019 A general variable neighborhood search-based solution approach for the location-inventory-routing problem with distribution outsourcing *Computers & Chemical Engineering* **126** 263–279

[7] Li K, Li D and Wu D 2020 Multi-objective optimization for location-routing-inventory problem in cold chain logistics network with soft time window constraint *Journal Europen des Systmes Automatiss* **53(6)** 803–809

[8] Sivasubramani S and Swarup K S 2011 Multi-objective harmony search algorithm for optimal power flow problem *International Journal of Electrical Power & Energy Systems* **33(3)** 745–752

[9] Sivasubramani S and Swarup K S 2011 Environmental/economic dispatch using multi-objective harmony search algorithm *Electric Power Systems Research* **81(9)** 1778–1785

[10] Luo Q, Wu J, Sun X, Yang Y and Wu J 2012 Optimal design of groundwater remediation systems using a multi-objective fast harmony search algorithm *Hydrogeology Journal* **20(8)** 1497–1510

[11] Li Y, Li X and Gupta J N 2015 Solving the multi-objective flowline manufacturing cell scheduling problem by hybrid harmony search *Expert Systems With Applications* **42(3)** 1409–1417

[12] Kavousi-Fard A, Abbasi A and Baziar A 2014 A novel adaptive modified harmony search algorithm to solve multi-objective environmental/economic dispatch *Journal of Intelligent & Fuzzy Systems* **26(6)** 2817–2823

[13] Geem Z W 2010 Multiobjective optimization of time-cost trade-off using harmony search *Journal of Construction Engineering and Management* **136(6)** 711–716

[14] Wang L, Mao Y, Niu Q and Fei M 2011 A multi-objective binary harmony search algorithm *In International Conference in Swarm Intelligence* Springer, Berlin, Heidelberg pp 74–81

[15] Salcedo-Sanz S, Manjarres D, Pastor-Sànchez À, Del Ser J, Portilla-Figueras J A and Gil-Lopez S 2013 One-way urban traffic reconfiguration using a multi-objective harmony search approach *Expert Systems With Applications* **40(9)** 3341–3350

[16] Landa-Torres I, Manjarres D, Salcedo-Sanz S, Del Ser J and Gil-Lopez S 2013 A multi-objective grouping harmony search algorithm for the optimal distribution of 24-hour medical emergency units *Expert Systems With Applications* **40(6)** 2343–2349

[17] Hajipour V, Rahmati S H A, Pasandideh S H R and Niaki S T A 2014 A multi-objective harmony search algorithm to optimize multi-server locationallocation problem in congested systems *Computers & Industrial Engineering* **72** 187–197

[18] Qu B Y, Li G S, Guo Q Q, Yan L, Chai X Z and Guo Z Q 2019 A niching multi-objective harmony search algorithm for multimodal multi-objective problems *IEEE Congress on Evolutionary Computation (CEC)* pp 1267–1274

[19] Zinati R F and Razfar M R 2019 Multi-objective constrained optimization of turning process via modified harmony search algorithm *Iranian Journal of Science and Technology, Transactions of Mechanical Engineering* **43(2)** 375–382

[20] Alinezhad A, Kazemi A and Karimi M 2020 Presenting a multi-objective model for location-routing problem with considering minimal risk and maximal demand covering *Industrial Management Studies* **18(58)** 105–138

[21] Geem Z W, Kim J H and Loganathan G V 2001 A new heuristic optimization algorithm: harmony search *Simulation* **76(2)** 60–68

[22] Yang X S 2009 *Harmony search as a metaheuristic algorithm. In music-inspired harmony search algorithm* Springer, Berlin, Heidelberg pp 1–14

[23] Zhang T and Geem Z W 2019 Review of harmony search with respect to algorithm structure *Swarm and Evolutionary Computation* **48** 31–43
[24] Mahdavi M, Fesanghary M and Damangir E 2007 An improved harmony search algorithm for solving optimization problems *Applied Mathematics and Computation* **188(2)** 1567–1579
[25] Mohd Alia O and Mandava R 2011 The variants of the harmony search algorithm: an overview *Artificial Intelligence Review* **36(1)** 49–68
[26] Misni F and Lee L S 2021 Modified harmony search algorithm for location-inventory-routing problem in supply chain network design with product returns *Malaysian Journal of Mathematical Sciences* **15(1)** 1–20
[27] Kirkpatric S 1983 Optimization by simulated annealing *Science* **220(4598)** 671–680
[28] Deb K, Pratap A, Agarwal S and Meyarivan T A M T 2002 A fast and elitist multiobjective genetic algorithm: NSGA-II *IEEE Transactions on Evolutionary Computation* **6(2)** 182–197
[29] Misni F, Lee L S and Seow H-V 2020 Hybrid harmony search-simulated annealing algorithm for location-inventory-routing problem in supply chain network design with defect and non-defect items *Applied Sciences* **10(18)** 6625